

# 视觉形状知觉在近似数量系统和计算流畅性关系中的作用

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**摘要** 已有大量研究揭示了近似数量系统与计算流畅性的相关关系, 但缺少对二者关系原因的系统检验与论证。视觉形状知觉假设有别于传统的数量领域特异性解释, 认为对形状的快速知觉是近似数量系统与计算流畅性的共同认知机制, 即视觉形状的快速知觉能力可以解释二者之间的相关关系。近似数量系统和计算流畅性在加工过程中依赖对形状的快速知觉, 二者在加工过程中都涉及了复杂视觉刺激的快速处理。视觉形状知觉假设得到了一系列研究结果的支持, 但局限在视觉形状知觉与二者关系的探讨上, 视觉形状知觉在二者关系中作用的加工机制仍不清楚。未来研究需要结合多种研究方法和技术, 多角度深入探讨视觉形状知觉在二者关系中作用的认知与脑机制, 并将研究结果应用于数学课堂教学和计算困难的干预中。

**关键词** 近似数量系统, 视觉形状知觉, 计算流畅性

## 1 前言

人们在日常生活中会遇到许多需要对数量进行迅速处理的情况, 这种不依赖语言和计数对数量信息进行加工的能力所依赖的认知系统被称为近似数量系统 (Approximate Number System, ANS) (Feigenson et al., 2004)。这种基础的数量加工能力是人类和其他物种共有的 (Dehaene et al., 1998; Hauser et al., 2000), 且在人类的婴儿期就已经出现 (McCrink & Wynn, 2004; Xu & Spelke, 2000; Xu, 2003)。计算流畅性是指个体又快又准确地解决简单符号化算术问题的能力, 是数学能力中的重要组成部分 (Carr & Alexeev, 2011; Geary et al., 1992)。计算

流畅性是数学问题解决的基础, 计算流畅性较高的个体可以节省更多的认知资源为复杂的问题解决和问题推理提供服务 (Meyer et al., 2010)。基础的数量加工能力与计算流畅性的关系是研究者们非常关注的问题。大量研究探讨了近似数量系统在计算流畅性中的重要性 (e.g., Bugden & Ansari, 2016; Cui, Zhang et al., 2019; De Smedt et al., 2013; Halberda et al., 2008; Hyde et al., 2014; Inglis et al., 2011; Lindskog et al., 2014; Mazzocco et al., 2011; Odic et al., 2016; Piazza et al., 2010; Park & Brannon, 2013, 2014; Price et al., 2007; Qu et al., 2021)。

围绕二者的相关关系, 越来越多的研究者开始关注近似数量系统为什么会在计算流畅性中发挥作用, 并尝试从中介角度对二者关系进行解释。但这些解释是多样的, 缺少足够的直接证据, 同时有部分实验证据在以往提出的假设下得不到合理的解释。近期有研究者提出“视觉形状知觉假设”, 认为对形状的快速知觉是近似数量系统与计算流畅性加工共同依赖的重要认知因素 (Cheng et al., 2018; Cheng et al., 2020; Cui et al., 2017; Cui, Zhang et al., 2019; Li et al., 2020; Wang et al., 2016; Zhang, Liu, & Zhou, 2019; Zhou et al., 2015; Zhou et al., 2020)。由此, 本文基于近似数量系统与计算流畅性的关系, 对比视觉形状知觉假设与其他假设的异同, 总结视觉形状知觉假设在近似数量系统和计算流畅性、以及二者关系中作用的研究成果, 探讨当前关于视觉形状知觉假设研究的局限, 并对未来研究做出展望。

## 2 近似数量系统与计算流畅性的关系

测查近似数量系统的常用范式是点阵数量比较 (e.g., Halberda et al., 2008; Piazza et al., 2010; Qu et al., 2021)。该任务会快速呈现两个点阵, 让被试在难以数清点个数的前提下判断哪个点阵包含的点数更多。混合点阵 (e.g., Halberda et al., 2008) 和分离点阵 (e.g., Zhou et al., 2015) 是点阵数量比较常用的刺激呈现模式。点阵数量比较的任务难度一般是由两个点阵中点的数量比例来控制, 比例越接近 1, 两组点阵在数量上越接近, 被试越难进行判断。

已有大量研究揭示了近似数量系统在计算流畅性中的重要性。这些结果最早体现在计算障碍群体上, 研究者发现计算障碍群体在点阵数量比较任务上要落后于正常发展组 (e.g., Bugden & Ansari, 2016; Mazzocco et al., 2011; Mussolin et al., 2010; Piazza et al., 2010; Price et al., 2007)。例如 Piazza 等人 (2010) 的研究发现计算障碍人群的 ANS 敏锐度比正常发展的同龄人更差。具体表现在与正常发展儿童相比, 计算障碍儿童在点阵数量比较任务上的反应时更长, 错误率更高 (Bugden & Ansari, 2016; Mussolin et al., 2010)。近似数量系统与计算流畅性的相关关系还体现在正常发展儿童和成人身上, 包括不同年级的学龄儿童 (Cui, Zhang

et al., 2019; Gilmore et al., 2010; Halberda et al., 2008; Lourenco et al., 2012; Qu et al., 2021; Zhang et al., 2016; Zhang, Liu, & Zhou, 2019) 以及成人 (Halberda et al., 2012; Libertus et al., 2012; Lindskog et al., 2014)。一项对 1857 名小学三到六年级学生进行的大样本研究表明, ANS 敏锐度与计算流畅性相关, 甚至独立于智力、空间加工能力、加工速度、数字大小比较能力的影响 (Zhang et al., 2016)。研究者也从认知神经科学角度验证了近似数量系统和计算流畅性的紧密关系。他们测试了 10 岁儿童的计算能力, 并在两年后测试了他们的 ANS 敏锐度, 发现十岁时计算越流畅的儿童, 在两年后测试 ANS 敏锐度时顶内沟的激活程度越大 (即 ANS 敏锐度越高) (Suárez-Pellicioni & Booth, 2018)。

此外, 训练研究发现基于近似数量系统的训练可以有效提高儿童和成人的计算成绩 (e.g., Hyde et al., 2014; Park & Brannon, 2013, 2014; Park et al., 2016)。例如, Hyde 等人 (2014) 对 6~7 岁儿童进行了点阵数量比较和点阵数量加法的训练, 这种训练使儿童的计算流畅性得到了提高。成年人在接受点阵的近似算术任务训练之后, 其计算成绩和 ANS 敏锐度都显著提高 (Park & Brannon, 2013, 2014), 并且所提高的计算成绩与 ANS 敏锐度的变化显著相关 (Park & Brannon, 2013)。

但也有部分研究并没有发现二者的相关关系 (Fuhs & McNeil, 2013; Holloway & Ansari, 2009; Kolkman et al., 2013; Lonnemann et al., 2011; Lyons & Beilock, 2011; Mundy & Gilmore, 2009; Price et al., 2012)。这可能受到点阵数量比较任务不同的测量范式和统计指标的影响 (Inglis & Gilmore, 2014)。例如, 当使用数量距离效应 (Numeric Distance Effect, NDE) 作为统计指标时, 近似数量系统和计算流畅性无关 (Holloway & Ansari, 2009; Lonnemann et al., 2011; Mundy & Gilmore, 2009)。研究者强调以正确率、反应时和韦伯分数作为点阵数量比较任务的统计指标更为合理, 结果更为稳定 (Schneider et al., 2017)。另外, 被试的年龄差异、是否控制一般认知能力等因素也会影响近似数量系统和计算流畅性的关系 (Chen & Li, 2014; Schneider et al., 2017)。例如, Wilkey 和 Ansari (2020) 在综述近似数量系统与数学能力的无关关系时指出, 个体在点阵数量比较任务中感知到的可能是点阵的连续视觉属性, 而不是数量信息; 另外, 点阵数量比较任务可能还包含了其他认知成分作用于数学加工, 例如对点阵视觉信息的抑制控制。

### 3 近似数量系统与计算流畅性关系的理论解释

围绕近似数量系统与计算流畅性的相关关系,越来越多的研究者开始关注近似数量系统为什么会在计算中发挥作用。这实质上是有关计算加工认知机制的问题。解决这一问题,将帮助我们理解近似数量系统在计算加工中的作用,也为通过近似数量系统的训练促进儿童计算流畅性发展提供理论基础。我们对以往文献进行总结,将这些理论划分为两部分,包括领域一般性解释和数量领域特异性解释。

#### 3.1 领域一般性解释

领域一般性解释认为一般性的认知因素可以解释近似数量系统在计算流畅性中的作用。目前主要有抑制控制假设 (Fuhs & McNeil, 2013; Gilmore et al., 2013) 和视觉形状知觉假设 (Zhou et al., 2015; Zhang, Liu, & Zhou, 2019)。

“抑制控制假设”认为抑制控制可能是影响 ANS 敏锐度与计算流畅性关系的重要因素 (Fuhs & McNeil, 2013; Gilmore et al., 2013)。ANS 敏锐度通常使用点阵数量比较任务测量。在该任务中,研究者们考虑到视知觉对点阵数量判断的影响,生成两种条件的点阵图:一致条件,即点阵的视觉特征与点数正相关;不一致条件,即点阵的视觉特征与点数负相关。其中的不一致条件需要被试抑制点阵的视觉特征,专注于数量信息做出判断,这与经典的 Stroop 范式非常相似。Stroop 范式常用来测量抑制控制能力,并且儿童的抑制控制能力与其计算能力密切相关 (Clark et al., 2010; Espy et al., 2004; Robinson & Dubé, 2013)。因此 Gilmore 等人 (2013) 提出了抑制控制假设,认为 ANS 敏锐度与计算流畅性的相关关系是因为抑制控制在其中起到了中介作用。Gilmore 等人 (2013) 使用了一项涉及到视觉形状加工的任务 (Korkman et al., 2007) 来测量 7~11 岁儿童的抑制控制能力。在这个任务中会先呈现一系列黑白圆圈和方块,要求儿童在一定时间内对其进行命名,然后再要求儿童对其进行相反的命名(即呈现正方形时,命名为“圆”)。结果发现抑制控制能力可以解释儿童的 ANS 敏锐度和计算流畅性之间的关系。

“视觉形状知觉假设”认为视觉形状知觉是近似数量系统与计算流畅性的潜在认知机制,即在控制了视觉形状知觉敏锐度后近似数量系统与计算流畅性不再相关 (Zhou et al., 2015; Zhang, Liu, & Zhou, 2019; Zhou et al., 2020)。该假设强调对形状的快速知觉在二者关系中的作用。具体来说,近似数量系统和计算流畅性在加工过程中均依赖对形状的快速知觉,二者

在加工过程中都涉及了复杂视觉刺激的快速处理。计算流畅性在加工过程中涉及到对数学符号（数学符号包括数字、运算符等）形状的快速知觉，对算术事实的快速搜索与提取。点阵数量比较在快速的点阵数量大小判断中依赖点和点之间的结构关系。例如点阵的凸包形状大小（外围的点构成的几何图形）、点密度（点与点之间的紧密程度）等视知觉属性会影响ANS敏锐度。因此，在点阵加工过程中，对点和线（不可见的）之间结构关系的视觉形状快速感知能力非常重要。同时在计算流畅性的加工过程中也依赖视觉形状知觉，它可能与从点阵或其他视觉对象的布局中提取数量信息有着相似的认知加工过程。事实上有关近似数量系统和不同数学能力关系的研究发现，点阵数量比较能力与计算流畅性有关，但与较复杂的、流畅性较低的数学能力，例如问题解决、估算等无关（Cui et al., 2017; Wang et al., 2016; Zhang et al., 2016）。

视觉形状知觉假设和抑制控制假设虽然都强调一般认知因素在近似数量系统和计算流畅性关系中的作用，但抑制控制假设强调抑制控制能力对二者关系的解释，而形状知觉假设强调形状的快速知觉，即对形状的知觉速度在二者关系中的作用。视觉形状知觉假设与抑制控制假设并不矛盾，Gilmore 等人（2013）所使用的抑制控制任务可能涉及到视觉形状知觉能力。但目前已有一些研究结果不支持抑制控制假设（Keller & Libertus, 2015; Malone et al., 2019）。总结他们所使用的任务发现，抑制控制多采用头-脚-肩膀-膝盖任务进行测量，较少涉及形状知觉的加工。并且有些与抑制控制假设相悖的实验证据没有控制与视觉形状知觉相关的一般认知因素。

### 3.2 数量领域特异性解释

数量领域特异性解释认为点阵的数量信息是可以直接被提取的，近似数量系统和计算流畅性都涉及到对数量的处理，即数量加工是近似数量系统和计算流畅性二者的共同加工机制（Halberda et al., 2008）。近似数量系统是计算流畅性能力获得的基础（e.g., Bethany et al., 2016; Chu et al., 2015; Lyons & Beilock, 2011; van Marle et al., 2014）。另外，近似数量系统和计算流畅性都能够激活大脑数量加工的核心脑区，顶内沟区域（Ansari & Dhital, 2006; Izard et al., 2008）。

表面上看视觉形状知觉假设与数量领域特异性假设是互相冲突的。但有两点值得注意，一方面，Halberda 等人（2008）的研究发现儿童的ANS敏锐度与他们的计算成绩相关，甚至在控制了大量的一般认知能力包括智力、视觉工作记忆、空间推理、阅读、执行功能等，



二者的相关关系仍然存在。但值得注意的是，除了点阵数量比较，该研究中所涉及的认知任务都不是快速呈现，即很少依赖对形状知觉的加工速度，这可能解释了近似数量系统对计算成绩独特的预测作用。另外，在支持数量领域异性假设的其他实验证据中 (Libertus et al., 2011, 2013; Mazzocco et al., 2011; Bonny & Lourenco, 2013; Chu et al., 2015; Starr et al., 2013; van Marle et al., 2014)，大多都缺少对加工速度的要求 (例如，早期数学能力测试 TEMA)，即被试不需要快速地对问题进行加工和回答。而视觉形状知觉假设强调近似数量系统和计算流畅性的紧密关系，以及快速的视觉形状知觉在二者关系中的解释作用。另一方面，这些揭示 ANS 敏锐度与计算流畅性相关关系的研究缺少足够的一般认知因素的控制，尤其是形状知觉的控制 (Libertus et al., 2011, 2013; Mazzocco et al., 2011; Bonny & Lourenco, 2013; Chu et al., 2015; Starr et al., 2013; van Marle et al., 2014)。例如，有研究者在控制了视觉加工能力后，ANS 敏锐度与计算不再相关 (Zhang et al., 2019)。

## 4 视觉形状知觉在近似数量系统和计算流畅性中的作用

基于以上总结发现，关于近似数量系统和计算流畅性相关关系的认知机制问题缺少足够的直接证据。虽然有实验结果直接或间接地支持了数量领域特异性和领域一般性的抑制控制假设，但仍有部分实验结果很难在该理论假设下得到合理解释 (Cui et al., 2017; Keller & Libertus, 2015; Malone et al., 2019)。近期有研究者围绕近似数量系统和计算流畅性的相关关系，直接提出并检验了“视觉形状知觉假设”。下面我们将总结视觉形状知觉在近似数量系统和计算流畅性中作用的直接和间接证据。

### 4.1 视觉形状知觉在计算流畅性中的作用

已有研究表明计算障碍儿童在形状知觉加工任务上表现出缺陷 (Rourke & Finlayson, 1978; Sigmundsson et al., 2010; Zhou & Cheng, 2015)。例如 Zhou 和 Cheng (2015) 在控制了计算障碍儿童的选择反应时、空间能力、视觉追踪能力和智力后，发现他们的视觉形状知觉能力 (使用快速图形匹配任务测量) 存在缺陷。与阅读障碍儿童和正常发展儿童相比，计算障碍儿童在包含形状加工的抑制控制任务中表现更差 (Wang et al., 2012)。正常发展儿童的视觉形状知觉加工能力与他们的计算成绩也存在相关关系，甚至在控制其他一般认知因素，例如智力和听知觉后二者仍然相关 (Cui, Zhang et al., 2019; Kurdek & Sinclair, 2001; Rosner,

1973)。例如, Cui 等人发现三至五年级儿童的视觉形状知觉能力是计算能力的潜在认知机制 (Cui, Zhang et al., 2019)。

此外, 枕叶也是计算加工的关键脑区 (Kuhl et al., 2020; Liu et al., 2017; Polspoel et al., 2017; Vansteensel et al., 2014; Wang et al., 2019)。Tinelli 等人 (2015) 发现枕叶损伤会影响符号化数字的辨别能力。因双侧枕叶受损, 而导致视觉形状失认症的患者 (D.F.) 无法区分简单的几何形状、阿拉伯数字和字母, 虽然其听觉计数能力正常, 但难以进行点的数量判断和简单计算 (Cavina-Pratesi et al., 2015; Milner et al., 1991)。相比依赖乘法口诀记忆的乘法计算, 加法运算更依赖数字视觉表象加工, 激活了更多视觉空间加工的脑区, 包括枕中回、枕上回、右侧顶内沟等区域 (Zhou et al., 2007)。

#### 4.2 视觉形状知觉在近似数量系统中的作用

已有研究证实在加工点阵的过程中会受到视知觉属性的影响, 包括累积表面积 (Guillaume et al., 2013)、平均点面积 (Henik et al., 2017)、密度 (Dakin et al., 2011) 以及凸包 (包含所有点的最小凸多边形) (Norris et al., 2018)。有研究者发现个体对点阵形状的加工是一种自动的整体加工 (Katzin et al., 2020; Picon et al., 2019; Tibber et al., 2012), 他们甚至会依赖视知觉属性来判断点阵数量的大小。这不仅体现在计算障碍儿童 (Bugden & Ansari, 2016) 中, 也体现在正常发展儿童 (Defever et al., 2013; Tokita & Ishiguchi, 2013) 和成人 (Clayton et al., 2015; Katzin et al., 2020) 中。例如, 计算障碍儿童在点阵视觉特征与数量不一致时, 很难对点阵数量做出判断 (Bugden & Ansari, 2016)。Clayton 等人 (2015) 发现对点阵视觉属性的控制水平会影响 ANS 敏锐度的测量。

近似数量系统加工的神经科学研究也体现了视觉形状知觉在点阵数量比较中的作用。Dewind 等人 (2019) 的 fMRI 研究发现个体在编码点阵数量信息时激活了早期视觉皮层。一项 ERP 研究在控制了点阵视觉特征之后, 发现个体是在权衡了数量刺激中存在的不同视觉线索之后对数量做出的判断 (Gebuis & Reynvoet, 2012)。Gebuis 和 Reynvoet (2014) 发现在视觉信息与数量信息一致条件下和不一致条件下个体的神经反应不同, 这一结果支持了近似数量加工会依赖视觉线索的观点。

#### 4.3 视觉形状知觉在二者关系中的作用

也有研究者直接证实了视觉形状的快速知觉是近似数量系统与计算流畅性共同依赖的认知因素。研究者采用快速图形匹配任务考察视觉形状知觉，揭示了视觉形状的快速知觉在近似数量系统和计算流畅性中的作用 (e.g., Cheng et al., 2018; Cheng et al., 2020; Cui, Zhang et al., 2019; Li et al., 2020; Wang et al., 2016; Zhang, Liu, & Zhou, 2019; Zhou & Cheng, 2015; Zhou et al., 2015; Zhou et al., 2020)。在该任务中，每个试次的左边都有一个目标图形，右边都有三张候选图形。每个试次呈现 400 毫秒。要求被试又快又准地判断左侧的目标图形是否出现在右侧的候选图形中（见图 1）。

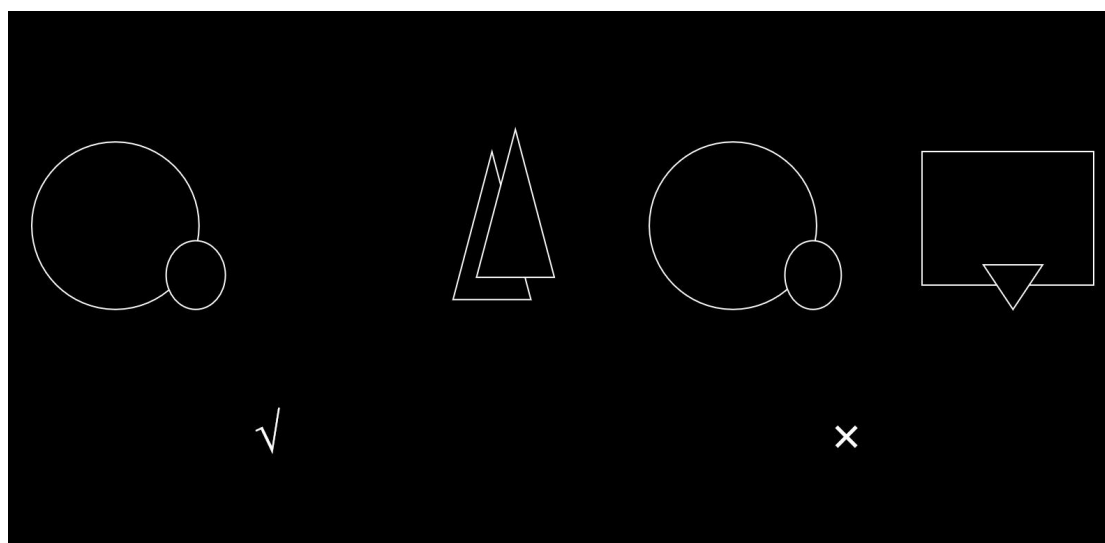


图 1 快速图形匹配任务示例 (图片来源: Zhou et al., 2015 *Frontiers in Psychology*.)

Zhou 等人 (2015) 对 424 名三至五年级小学生的研究发现，在控制了非语言矩阵推理、选择反应时、视觉追踪能力、工作记忆和心理旋转后，快速图形匹配任务测量的视觉形状知觉解释了 ANS 敏锐度和计算流畅性之间的关系。后续一系列研究也证实了视觉形状知觉在近似数量系统和计算流畅性中的作用 (Cui et al., 2017; Cui, Zhang et al., 2019; Wang et al., 2016; Zhou et al., 2020)。例如，小学三年级的视觉形状知觉可以预测他们三年后的计算成绩 (Zhou et al., 2020)。且这种视觉形状知觉加工能力不仅是计算流畅性也是阅读理解能力的认知基础 (Cui, Zhang et al., 2019)。另外，有研究发现只有包含形状知觉较多的视觉加工能力（快速视知觉、视觉短时记忆）可以解释近似数量系统和计算流畅性的关系，而包含形状知觉较少的空间加工能力（三维心理旋转、空间短时记忆）并不能解释二者的关系 (Zhang, Liu, & Zhou, 2019)。

计算障碍的相关研究也直接支持了视觉形状知觉在近似数量系统与计算流畅性间的作用 (Cheng et al., 2018; Cheng et al., 2020; Zhou & Cheng, 2015)。研究发现，阅读障碍儿童和



计算障碍儿童的视觉形状知觉能力要落后于正常发展儿童,他们在视觉形状知觉上的不足导致了ANS敏锐度以及计算和阅读成绩的落后 (Cheng et al., 2018)。对ANS敏锐度的训练提高了计算障碍儿童的视觉形状知觉敏锐度,进而提高了他们的计算流畅性 (Cheng et al., 2020)。

一项ERP研究为视觉形状知觉假设提供了新的证据。Li等人 (2020) 发现负责视觉形状知觉能力的枕部区域可能支持ANS敏锐度和计算流畅性之间的联系。这项研究发现大脑左右侧的枕叶部位电极的N1成分与简单加减法、点阵数量比较和快速图形匹配任务的反应时显著相关。此外,在简单加减法加工过程中,枕叶部位电极的N1成分与快速图形匹配和点阵数量比较任务的反应时间也相关。

## 5 小结与展望

综上所述,本文以近期所提出的“视觉形状知觉假设”为核心,基于近似数量系统与计算流畅性的相关关系,总结了视觉形状的快速知觉在二者关系中的解释作用,是近似数量系统和计算流畅性共同依赖的认知因素。“视觉形状知觉假设”的提出有别于传统的数量领域特异性假设,认为近似数量系统可能在知觉层面上对计算加工过程产生作用,而不是领域特异的数量加工。这提示教育工作者在数学教育教学中,要更加重视学生基础认知能力的发展,要加强对数学符号形状表征与加工能力的培养。

但目前有关视觉形状知觉假设研究仍有些局限。首先,对于视觉形状知觉假设的研究方法较为单一,大多都是行为研究,仅有一项研究采用了事件相关电位法,从锁时角度来检验视觉形状知觉假设,缺少更加精准的脑区域定位研究。其次,以往验证视觉形状知觉假设的研究中,测量视觉形状知觉的任务较单一,多采用快速图形匹配任务,无法排除任务特异性对实验结果的影响。再次,以往研究所使用的视知觉形状加工任务可能混淆了其他一般认知因素,例如抑制控制、视觉注意的影响,我们并不清楚到底是视觉形状的快速知觉发挥了作用,还是连同抑制控制和视觉注意一起发挥了作用。另外,以往直接检验视觉形状知觉假设的证据集中在近似数量系统和计算流畅性的相关关系上,而计数、概念理解、规则学习、问题解决等都是重要的数学能力,需要得到进一步检验。最后也是最重要的一点,目前研究局限在视觉形状知觉与近似数量系统和计算流畅性关系的探讨上,关于视觉形状知觉为什么会在二者关系中发挥作用,即视觉形状知觉在二者关系中作用的加工机制仍不清楚,缺少理论体系的支持和实验数据的直接检验。

因此,未来研究需要结合多种手段对视觉形状知觉在近似数量系统和计算流畅性中作用的认知与脑机制进行深入探讨。具体包括:

第一,未来研究可结合认知行为测试、神经影像学技术,多角度探索形状的知觉速度在不同数学加工中的作用。基于视觉形状知觉假设发现,近似数量系统与计算流畅性紧密相关,形状的快速知觉可能是视觉形状知觉在二者关系中发挥作用的原因。事实上,视知觉任务范式(快速图形匹配)也是知觉速度的经典测量任务(Ekstrom et al., 1976)。根据 Cattell-Horn-Carroll (CHC) 智力模型,知觉速度是智力的基础和重要组成成分(Schneider & McGrew, 2012),在计算中发挥着重要作用(Bull & Johnston, 1997; Corso & Dorneles, 2014; Salthouse & Coon, 1994)。未来研究可以采用相关研究和实验设计,利用中介分析、操控影响变量(例如,加工速度,形状加工的复杂度)来揭示视觉形状知觉在近似数量系统和计算流畅性中作用的认知机制。以往研究发现计算和数学规则依赖的脑区不同,数学规则更依赖额叶和颞叶的联结区域(Desco et al., 2011);计算加工更多地是对数学符号(阿拉伯数字)的提取,更依赖顶枕叶联结(Liu et al., 2017; Zhang, Wee et al., 2019)。基于此,未来研究可以从神经层面上探索视觉形状知觉在近似数量系统和不同符号化数学能力中作用的脑机制。

第二,未来研究还需要考虑其他一般认知因素,例如抑制控制、视觉注意等对二者关系的影响,深入探讨到底是什么认知成分在近似数量系统和计算流畅性关系中发挥作用。已有研究揭示了抑制控制在计算(Clark et al., 2010; Espy et al., 2004; Robinson & Dubé, 2013)和点阵数量比较(Fuhs et al., 2016; Gilmore et al., 2013; Viarouge et al., 2019)中的重要性。除了抑制控制,视觉注意也可能影响视觉形状知觉在二者关系中的作用,因为注意能力是这三种能力测试(快速图形匹配、点阵数量比较和计算)所必需的。当前研究并不清楚到底是视觉形状的快速知觉发挥了作用,还是连同抑制控制或视觉注意一起发挥了作用。因此,未来研究需要层层剥离出不同认知因素对视觉形状知觉假设的影响。

第三,未来研究可以在不同文化背景下深入探讨视觉形状知觉与近似数量系统和计算流畅性的关系,以及在二者关系中作用的加工机制问题。与西方文化所依赖的字母语言不同,中国的汉字包括表象文字中的字符和笔画的组合,更加依赖形状知觉,汉字的字形和视觉复杂度会影响人们对汉字的识别(曾捷英等, 2001; 周新林, 曾捷英, 2002; Kuo et al., 2014)。而字母语言通常仅涉及了单词和字母的发音。如果形状知觉是计算加工的关键,在形状复杂度不同的文化语言影响下学习者的计算流畅性是否也不尽相同?中国数学教材都使用了大量的汉字来解释数学知识和概念,基于视觉形状知觉假设,中国儿童对汉字的学习经验可能会有助于他们对数学的学习,这可能会使中国儿童在计算加工中更多的依赖视觉形状知觉。

因此未来研究需要从跨文化角度对比中西方儿童的计算流畅性,对视觉形状知觉假设进行检验和深入分析。

第四,心理研究的最终目的是为了能够更好地服务于社会,造福人类。未来研究还需要在实践应用中检验视觉形状知觉假设。从因果关系上来论证视觉形状知觉在数学加工中的有效性,对计算障碍儿童进行有效干预研究,以及基于视觉形状知觉训练促进正常群体的数学加工能力。在数学教育教学情境中,如何基于视觉形状知觉加工的作用来培养学生的形状表征能力和数学学习能力也是值得深入思考和探究的。

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# Role of visual form perception in the relationship between approximate number system and arithmetical fluency

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**Abstract:** A large number of studies have revealed the correlation between the approximate number system and arithmetical fluency, but systematic tests and arguments for the causes of the relationship are lacking. The hypothesis of visual form perception differs from the traditional domain-specific explanation of number, and it is believed that the fast perception of forms is a common cognitive mechanism of the approximate number system and arithmetical fluency, that is, the fast perceptual ability of visual forms can explain the correlation between the two. Both the approximate number system and arithmetical fluency rely on the fast perception of forms and involve fast processing of complex visual stimuli during processing. The hypothesis of visual form perception is supported by the results of a series of studies but is limited to the exploration of the relationship between visual form perception and the approximate number system and arithmetical fluency, where the processing mechanisms underlying the role of visual form perception in the relationship between the two remain unclear. Therefore, future research needs to combine multiple research methods and techniques to comprehensively explore from multiple perspectives the cognitive and brain mechanisms underlying the role of visual form perception in the relationship between the two and to apply the findings to mathematics classroom teaching and interventions for dyscalculia.

**Keywords:** Approximate number system, Visual form perception, Arithmetical fluency